

Seat No. : _____

ND-113

November-2017

B.C.A., Sem.-II

CC – 111 : Mathematical Foundation of Computer Science

Time : 3 Hours]

[Max. Marks : 70

1. (A) Let a set $G = \{(a, b, c) / a, b, c \in \mathbb{Z}\}$ with operation addition defined on it as follows: **8**

For any $\alpha = (a_1, b_1, c_1), \beta = (a_2, b_2, c_2) \in G, \alpha + \beta = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$.

- (a) Show that $(G, +)$ is a group.
(b) Is it cyclic group ? Justify your answer.

OR

Let a set $G = \{2^n / n \in \mathbb{Z}\}$ with operation multiplication defined on it.

- (a) Show that G is a cyclic group.
(b) Show that $H = \{4^n / n \in \mathbb{Z}\}$ is a subgroup of G .

- (B) Let a set $G = \{1, -1, i, -i\}$ with operation multiplication defined on it. **6**

- (a) Show that G is an abelian group.
(b) Show that identity element of any group is unique.

OR

Let a set $G = \{0, 1, 2, 3, 4\}$ with operation addition reduced modulo 5.

- (a) Show that G is a group of order 5.
(b) Find the order of each element of the group $(G, +_5)$.

2. (A) Let a set $X = \{1, 2, 3, 4\}$ and relations on it as follows : **8**

$R_1 = \{(1, 1), (1, 3), (2, 3), (2, 4)\}$

$R_2 = \{(1, 1), (2, 2), (2, 3), (3, 2), (3, 3), (3, 4), (4, 3), (4, 4)\}$

Answer the following questions :

- (a) Give Domain and Range of the relation R_1 .
(b) Is the relation R_1 irreflexive ? Justify.
(c) Is the relation R_1 symmetric ? Justify.
(d) Write $R_3 = R_1 \cap R_2$. Is the relation R_3 reflexive ?

OR

Let Z be the set of integers and define a relation $R = \{(x, y) / x - y \text{ is divisible by } 3\}$ on Z .

- (a) Show that given relation R is an equivalence relation.
 - (b) Show that the set of all equivalence classes is a partition of Z .
- (B) Let a set S_{60} (S_m is a set of divisors of m) with relation D (divides) defined on it. 6
- (a) Show that is a Poset with respect to given relation.
 - (b) Draw the Hass diagram of the Poset $\langle S_{60}, D \rangle$.

OR

Let a Poset $\langle S_{45}, D \rangle$, where S_m = set of divisors of m and D is a partial ordering relation divides.

- (a) Find lower and upper bounds for the subset $\{5, 9, 15\}$ of S_{45} .
 - (b) Find minimal and maximal elements of a Poset S_{45} .
3. (A) Let a Poset $\langle P(A), \subseteq \rangle$, where $A = \{a, b, c\}$ and \subseteq is an inclusion relation on $P(A)$. 8
- (a) Show that a Poset $\langle P(A), \subseteq \rangle$ is a lattice with respect to g.l.b. and l.u.b. defined as intersection (\cap) and union (\cup) respectively on it.
 - (b) Show that a lattice $\langle P(A), \cap, \cup \rangle$ is a Boolean algebra with respect to complement defined as complement of a set.

OR

Let a Poset $\langle S_{60}, D \rangle$ with partial ordering D (divides).

- (a) Show that $\langle S_{60}, D \rangle$ is a lattice with respect to g.l.b. and l.u.b. defined on it as, g.l.b. $(a, b) = G.C.D.(a, b)$, l.u.b. $(a, b) = L.C.M.(a, b)$.
 - (b) Is a lattice $\langle S_{60}, g.c.d., l.c.m. \rangle$ complemented lattice ? Justify your answer.
- (B) Let a Boolean Algebra $\langle P(A), \cap, \cup, ', \emptyset, A \rangle$, where $A = \{1, 2, 3\}$. 6
- (a) Show that any pair of elements of $P(A)$ satisfies De Morgan's law.
 - (b) Show that a subset $\{\emptyset, \{a\}, \{b\}, \{a, b\}\}$ is a sub-Boolean algebra of $P(A)$.

OR

List all max terms in three variables x_1, x_2 and x_3 . Write the Boolean expression $x_1 * x_2$ in an equivalent sum of product canonical form in three variables x_1, x_2 and x_3 .

4. (A) Answer the following questions from given graph. (Figure - 1)

8

- Find reachable set of $\{v_1, v_8\}$.
- Give any two properties of node base.
- Find node base of given graph.
- Find the subgraph H of G with $V(H) = V(G) - \{v_1, v_{13}\}$

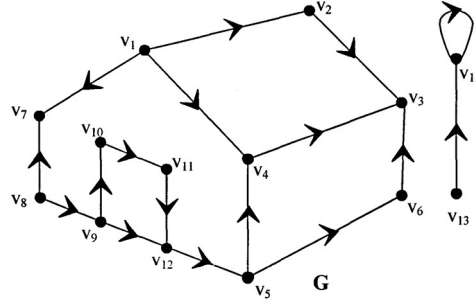


Figure - 1

OR

Draw the graph of each adjacency matrices given below. Are the simple graphs with the following adjacency matrices isomorphic ? Justify your answer.

$$M(G_1) = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \text{ and } M(G_2) = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

- (B) Give other representation of the Tree expressed by,
 $(v_0(v_1(v_4)(v_5)(v_6(v_{11}))))(v_2(v_7)(v_8))(v_3(v_9(v_{12}))(v_{10})))$

6

OR

Answer the following questions from the graph given below :

- Give a geodesic path from the vertex ' v_8 ' to the vertex ' v_1 '.
- Find the distance between two vertices v_1 and v_3 .
- Give the reachable set of a set v_1 .

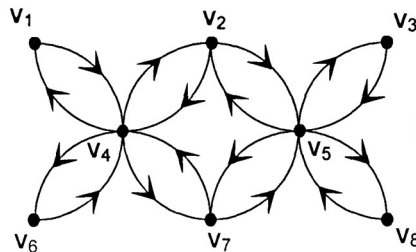


Figure - 2

3

- (1) Every cyclic group is an abelian group. (True / False)
- (2) Every group has a subgroup generated by an element of that group. (True / False)
- (3) Inverse of an element of a group is unique. (True / False)
- (4) If the order of a group is a prime number p then it has at least p subgroups. (True / False)
- (5) If a relation is not reflexive then it is irreflexive. (True / False)
- (6) If a relation is symmetric then its relation matrix is also symmetric. (True / False)
- (7) Every Poset need not be a lattice. (True / False)
- (8) Every subset of a lattice is a sublattice. (True / False)
- (9) Every Boolean Algebra is a lattice. (True / False)
- (10) A Poset P is a lattice if for any $a, b \in P$, _____.
 - (a) $a * b \in P$
 - (b) $a * b = 0$
 - (c) $(a * b)' = a' \oplus b'$
 - (d) None of these
- (11) _____ is a sublattice of a lattice $\langle S_{12}, D \rangle$.
 - (a) $\langle S_6, D \rangle$
 - (b) $\langle S_{30}, D \rangle$
 - (c) $\langle S_{45}, D \rangle$
 - (d) None of these
- (12) A relation on a set is an equivalence relation if that relation is _____.
 - (a) Reflexive
 - (b) Symmetric
 - (c) Transitive
 - (d) None of these.
- (13) Let a bounded lattice $\langle L, *, \oplus, 0, 1 \rangle$. An element $b \in L$ is called a complement of an element $a \in L$ if _____.
 - (a) $a * b = a$ and $a \oplus b = b$
 - (b) $a * b = b * a$ and $a \oplus b = b \oplus a$
 - (c) $a * b = 0$ and $a \oplus b = 1$
 - (d) None of these
- (14) In a Boolean Algebra $\langle B, *, \oplus, ', 0, 1 \rangle$, for any $a, b \in B$, $(a * b)' =$ _____.
 - (a) a
 - (b) 0
 - (c) 1
 - (d) None of these
